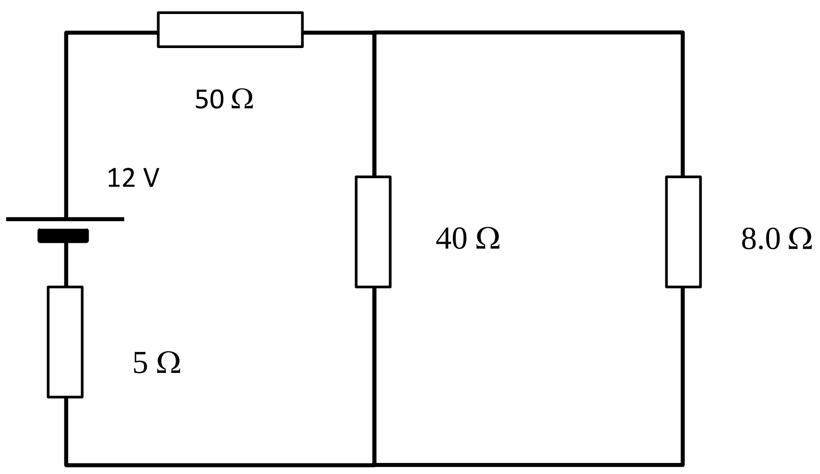
Leeds City College

Marking Scheme

Answers to Semester 2 Examination on Electrical Principles

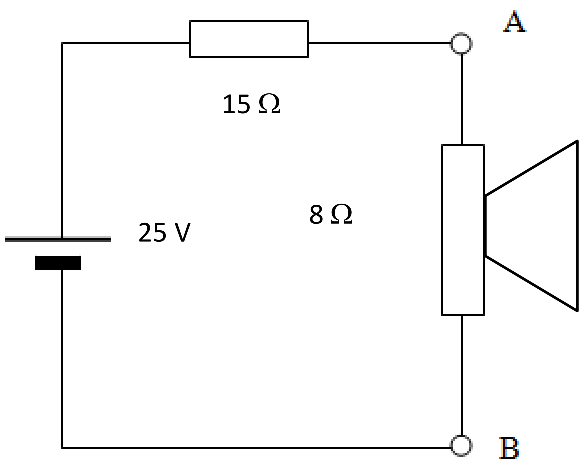
Examination Time – 120 minutes

- *Marking points are indicated by ticks (✓).*
- *Credit is awarded for each step.*
- *Credit may be awarded for correct or valid answers that are not on this marking scheme.*
- *A correct answer with no working gains the allotted mark minus one.*
- *Arithmetical errors are denoted with a.e.*
- *Physics errors are denoted with p.e and marking for that part ceases.*
- *All errors can be carried forward (e.c.f.).*
- *Where an error is carried forward, full credit can be awarded for the correct use of the error.*
- *Unit errors are penalised once only.*
- *Significant figure errors are penalised once only.*
- *Missing material is denoted by carets (^).*
- *Unattempted questions should be crossed out with a diagonal straight line.*

1.	Electrical engineers use the concepts of ideal constant voltage source and constant current sources to help them to analyse circuits			
	(a)	State what is meant by: (i) A constant voltage source; (ii) A constant current source.	**4.2**	(4)
	(i) A constant voltage source can maintain a specific voltage…			✓
	…regardless of the current drawn.			✓
	(ii) Ideal constant current source gives out a specific current…			✓
	…regardless of the voltage.			✓
	(b)	In reality circuits are not ideal. Leon Thévenin devised simple a rule that enabled electrical engineers to analyse circuits that did not have idea voltage sources. Write down Thévenin's Theorem.		(2)
	Any source however complex can be represented as…			✓
	…a simple perfect battery in series with an internal resistor.			✓
	(c)	The diagram shows a circuit:  Show that the Thévenin Voltage is about 5 V		(4)
	(Remove the 8 Ω resistor)			
	Total resistance = 95 Ω			✓
	I = 12 ÷ 95 = 0.126 A			✓
	V_{Th} = 0.126 × 40			✓
	V_{Th} = 5.1 V (≈ 5 V)			✓

	(d)	Show that the Thévenin resistance is about 23 Ω		(4)
	(Remove the batter and short out the gap, then peer in)			✓
	Work out the series resistors = 50 + 5 = 55 Ω			✓
	Work out the parallel resistors = $55^{-1} + 40^{-1} = 0.431$			✓
	R_{Th} = 23.2 Ω (≈ 23 Ω)			✓
	(e)	Work out the current in the 8.0 Ω resistor.		(3)
	R + r = 23.2 + 8 = 31.2 Ω			✓
	I = 5.1 ÷ 31.2			✓
	I = 0.16 A			✓
	(f)	Work out the power dissipated in the 8.0 ohm resistor.		(3)
	P = $0.163^2 \times 8$			✓
	P = 0.21 W			✓
	(g)	A student says that the maximum **useful** power transfer occurs when the current is at a maximum (i.e. when the source is shorted out). Another student says that it occurs when the outside load resistance is the same as the Thévenin resistance. A third student suggests that the power transfer is much higher when the outside resistance is higher than the Thévenin resistance. Using R = 0, 23 Ω and 50 Ω, demonstrate **with suitable supporting calculations** if necessary which student is correct.	**4.5**	(7)
	At short circuit, the useful power transfer is zero.			✓
	When R = r, V = 5.05 ÷ 2 = 2.53 V			✓
	P = $2.525^2 \div 23.2 = 0.27$ W			✓
	When R > 23 Ω, P is reduced.			✓
	R = 50 Ω, therefore R_T = 73.2 Ω			✓
	V = (50/73.2) × 5.05 = 3.45 V			✓
	P = $3.45^2 \div 50 = 0.24$ W			✓
	Maximum power transfer occurs when R = 23 Ω.			
				30 marks

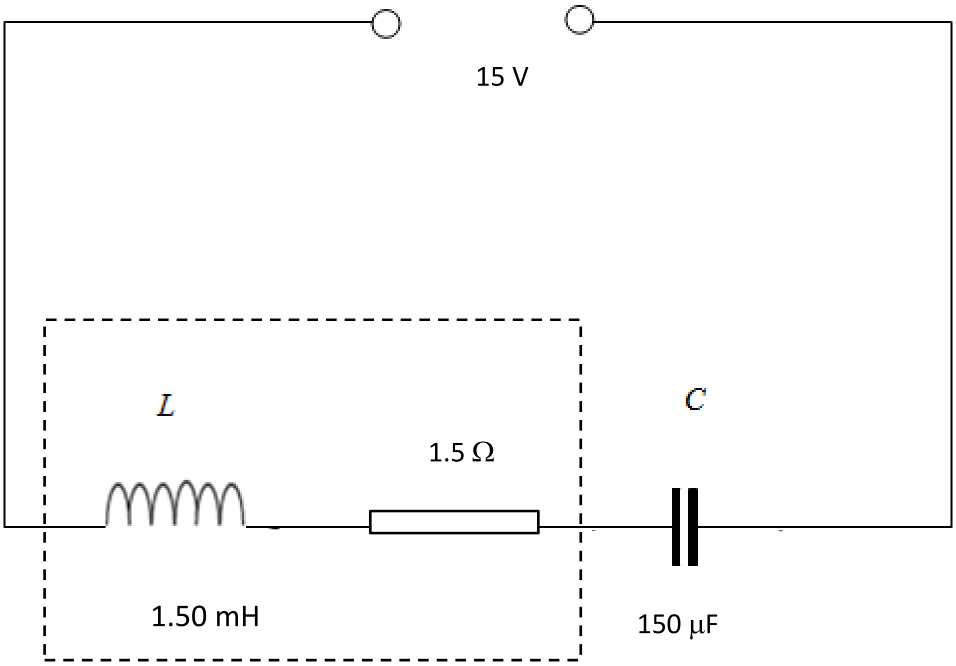
2. The diagram shows an amplifier that is modelled using Thévenin's theorem.



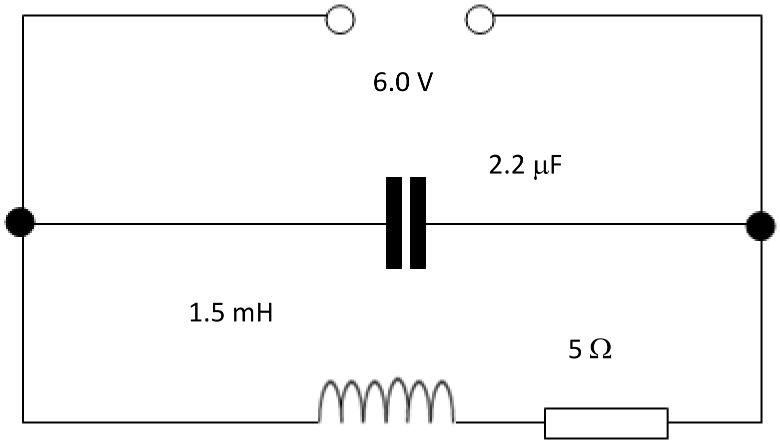
The output voltage is 25 V and the loudspeaker has an impedance of 8 ohms.
The internal impedance of the amplifier is 15 ohms.

	(a)	Explain why the term impedance is used instead of resistance.	(3)
	Alternating signals are used.		✓
	There are reactive components within the amplifier and loudspeaker…		✓
	…for example inductance in the loudspeaker.		✓
	In this question we are going to model the circuit as a DC source of potential difference 25 V and of internal resistance 15 ohms driving a load of 8 ohms		
	(b)	Show that the current in the circuit is about 1.1 A.	(2)
	R + r = 15 + 8 = 23 ohms		✓
	I = 25 V ÷ 23 Ω = 1.09 A (≈ 1.1 A)		✓
	(c)	Calculate the power given out by the speaker.	(2)
	$P = 1.087^2 \times 8$		✓
	P = 9.45 W		✓

	(d)	Calculate the total power given out by the source		(2)
	P = 25 × 1.087			✓
	P = 27.2 W			✓
	(e)	Work out the efficiency of the amplifier when driving an 8 ohm load.		(2)
	Efficiency = (9.45 ÷ 27.2) × 100 %			✓
	Efficiency = 35 %			✓
	(f)	Theory tells us that the maximum power transfer occurs when the internal resistance and the load resistance are both equal. This can be achieved by the use of a load-matching transformer. What should the resistance of the primary and secondary be to achieve this?	**4.6**	(2)
	Primary: 15 Ω			✓
	Secondary: 8 Ω			✓
	(g)	Work out the turns ratio for the matching transformer	**4.6**	(2)
	Ratio = $(15 \div 8)^{0.5}$			✓
	Ratio = 1.37:1			✓
	(h)	The output transformer has a primary of 2000 turns. What is the number of turns on the secondary?	**4.6**	(2)
	Secondary turns = 2000 ÷ 1.37			✓
	Secondary turns = 1460			✓
	(i)	Show that the power transferred to the loudspeaker is now about 10 W. State what assumption you have made.	**4.6**	(4)
	Total resistance is now 30 ohms			✓
	Current = 25 ÷ 30 = 0.833 A			✓
	Power = $0.833^2 \times 15 = 10.4$ W (≈ 10 W)			✓
	Assuming that there are no losses in the transformer.			✓
				21 marks

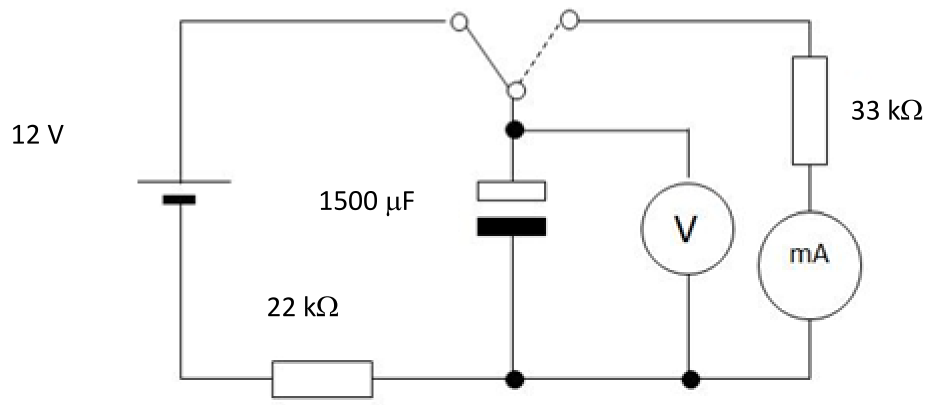
3	An inductor of inductance 150 mH and of resistance 1.5 Ω is placed in series with a capacitor of 150 μF, as shown in this diagram: 			
	(a)	How do you model an inductor with a known resistance?		(2)
	A perfect inductor…			✓
	… in series with a resistor.			✓
	(b)	Show that the resonant frequency for this combination is about 340 Hz		(2)
	$f = 1 \div (2 \times \pi \times (1.50 \times 10^{-3} \times 150 \times 10^{-6})^{0.5})$			✓
	$f = 1 \div (2 \times \pi \times (2.25 \times 10^{-6})^{0.5})$			
	f = 336 Hz (≈340 Hz)			✓
	(c)	Calculate the reactance of the inductor at resonance frequency. Give your answer to an appropriate number of significant figures.		(3)
	$X_L = 2 \times \pi \times 336 \times 1.50 \times 10^{-3}$			✓
	$X_L = 3.16\ \Omega$			✓
	Answer to 3 significant figures as data are to 3 s.f.			✓
	(d)	Write down the definition of the Q factor	**5.4**	(1)
	The ratio between the inductor voltage and the supply voltage			✓

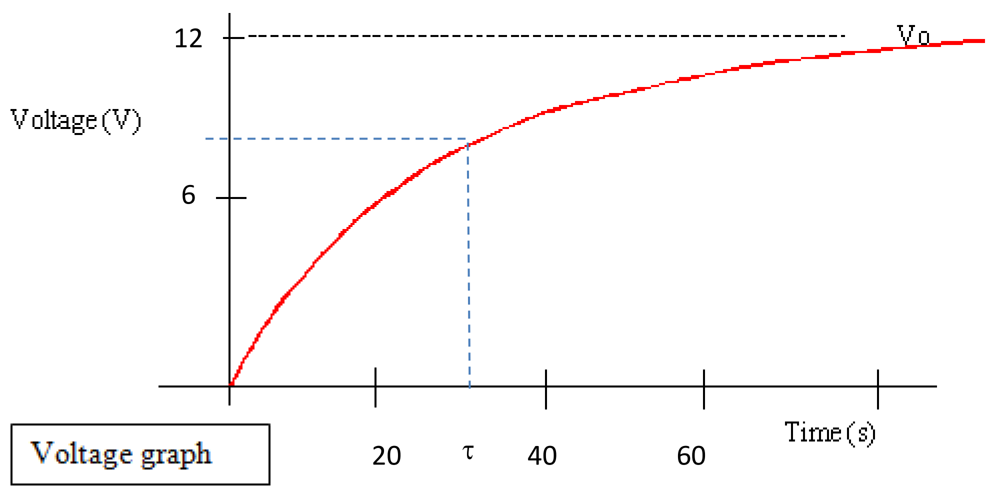
	(e)	Work out the Q factor for this circuit	**5.4**	(2)
	$Q = (2 \times \pi \times 336 \times 1.50 \times 10^{-3}) \div 1.50$			✓
	$Q = 2.1$			✓
	(d)	What is the voltage across the inductor?	**5.4**	(2)
	$V_L = 2.11 \times 15$			✓
	$V_L = 31.7$ V			✓
				12 marks

4.	The circuit below is used to investigate parallel resonance: 			
	(a)	Show that the resonant frequency of this circuit is about 2700 Hz. (Use the "exact" formula.)	**6.4**	(3)
	$f_0^2 = (1/4\,\pi^2) \times ((1.5 \times 10^{-3} \times 2.2 \times 10^{-6})^{-1} – (5^2 \div (1.5 \times 10^{-3})^2))$			✓
	$f_0^2 = (1/4\,\pi^2) \times ((303 \times 10^6) – (11.1 \times 10^6)$			
	$f_0^2 = (1/4\,\pi^2) \times (292 \times 10^6)$			✓
	$f_0^2 = 7.40 \times 10^6$			
	$f_0 = 2720$ Hz			✓
	(b)	A student looks at the formula you used to calculate your answer above, and decides to use the simpler formula below instead to work out the resonant frequency: $$f = \frac{1}{2\pi\sqrt{(LC)}}$$ (i) What is the resonant frequency given by this formula? (ii) Under what circumstance is the use of this formula justified? (iii) What is the percentage error in the approximation?	**6.4**	(5)
	(i) $f_0 = (2 \times \pi \times (1.5 \times 10^{-3} \times 2.2 \times 10^{-6})^{0.5})^{-1}$			✓
	$f_0 = 2770$ Hz			✓
	(ii) When R is small (compared to the reactances)			✓
	(iii) The result is 50 ohms too large.			✓
	This represents an error of (50 ÷ 2720) × 100 = 1.84 %			✓

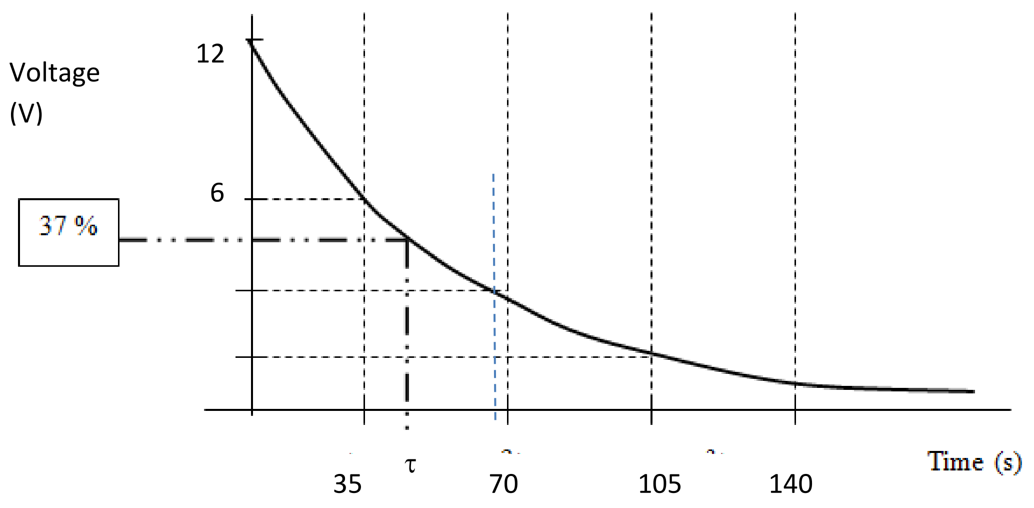
	(c)	Explain what happens when there is resonance and explain how the circuit above can act as a stable frequency source.	**6.6**	(4)
	At resonance, energy is exchanged…			✓
	…between the capacitor and inductor.			✓
	The impedance is very high.			✓
	So the current is very low.			✓
	The circuit gives a particular frequency…			✓
	…that is independent of voltage and current.			✓
	(*any 5 for 1 mark each*)			
	(d)	Give one other use for a resonant circuit.		(1)
	Radio receiver OR Electrical filter.			✓
	13 marks			

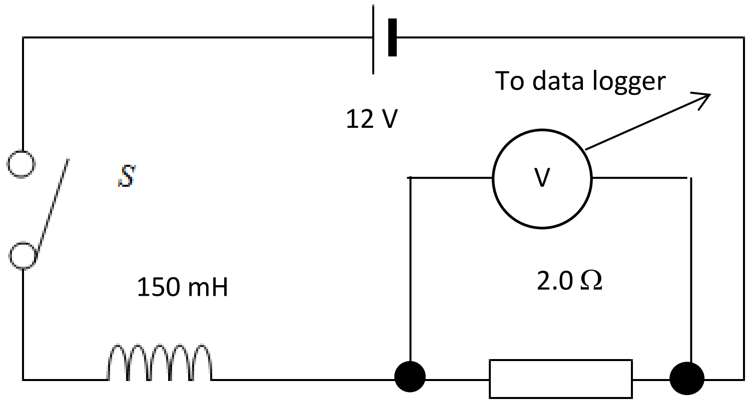
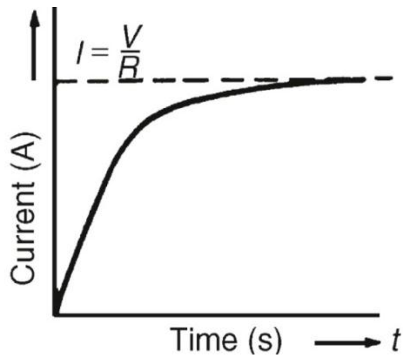
5.	The circuit below is being used to show the exponential charge or discharge of a capacitor through a resistor connected to a d.c. supply. The circuit as shown represents the **charging** of the capacitor.			
	(a)	One reading you can take for both charging and discharging is time (which you do with a stopwatch). What one other reading can you take while the capacitor is charging?	**8.1**	(1)
	The voltage			✓
	(b)	Explain why reactance is not a relevant quantity in this question.	**8.1**	(1)
	The capacitor is connected to a DC supply.			✓
	(c)	Define the time constant for the **charge** of the capacitor. Work out the time constant.	**8.3**	(3)
	Time taken for the voltage to reach 63 % of the final voltage			✓
	$\tau = 22\,000 \times 1500 \times 10^{-6}$			✓
	$\tau = 33$ s			✓
	(d)	Sketch the graph of voltage against time for the charge of the capacitor. Put suitable values on your sketch graph. Mark on the time constant.	**8.2**	(3)
				✓
				✓
				✓



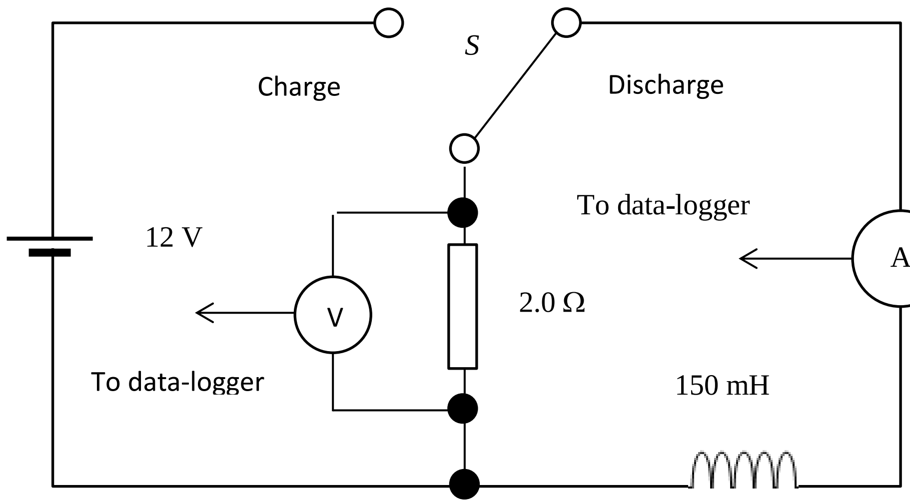


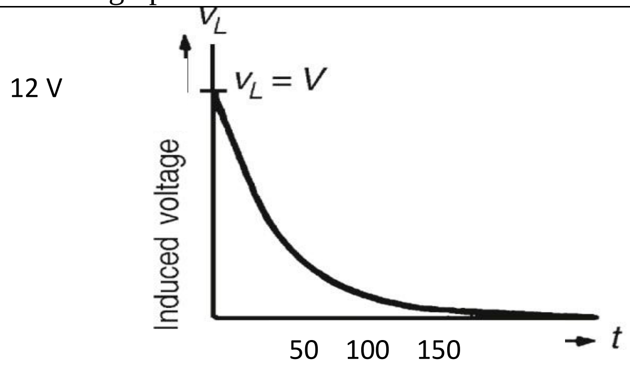
	The capacitor is now **discharged** through the 33 kΩ resistor			
	(e)	Calculate the time constant for the discharge.	**8.3**	(2)
	$\tau = 33\,000 \times 1500 \times 10^{-6}$			✓
	$\tau = 49.5$ s			✓
	(f)	Calculate the current at the start of the discharge	**8.4**	(2)
	$I = 12 \div 33000$			✓
	$I = 3.64 \times 10^{-4}$ A (= 0.36 mA)			✓
	(g)	Calculate the current after 30 s of the discharge.	**8.4**	(3)
	$I = 0.364 \times e^{(-30/49.5)}$			✓
	$I = 0..364 \times 0.545$			✓
	$I = 0.20$ mA			✓
	(d)	Sketch the graph of voltage against time for the discharge of the capacitor. Put suitable values on your sketch graph. Mark on the time constant.	**8.2**	(3)
	(graph below)			✓ ✓ ✓
	18 marks			



6.	A circuit consists of a pure inductor that is connected in series with a resistor as shown below.  Voltage is measured across the 2.0 Ω resistor. Switch S is closed.			
	(a)	Why is a data logger used?	**8.5**	(1)
	The time for the rise in voltage is very short.			✓
	(a)	Calculate the energy held in the inductor while the current is flowing through it.	**8.5**	(3)
	$I = 12 \div 2 = 6$ A			✓
	$E = ½ \times 150 \times 10^{-3} \times 6^2$			✓
	$E = 2.7$ J			✓
	(b)	When switch *S* is opened, the current falls to zero in 10 ms. Work out the EMF induced when switch S is opened.	**8.7**	(2)
	$E = (-)\ 0.150 \times (6 \div (-)1 \times 10^{-2})$			✓
	E = 90 V (enough to give you a tingle)			✓
	(c)	Switch *S* is closed again. The voltage across the 2.0 ohm resistor is measured using a fast data-logger. An inductive rise is observed. Sketch a graph of the voltage against time that you would expect to see. No values are needed. Explain why it is this shape.	**8.5** **8.6**	(4)
	 			✓ ✓

	Magnetic field in the inductor is building up…			✓
	…and produces an opposing EMF.			✓
	Rise is exponential.			
	(d)	Show that the time constant for this inductor and resistor is 75 ms	**8.3**	(2)
	$\tau = 0.15 \div 2$			✓
	$\tau = 0.075$ s (= 75 ms)			✓
	(e)	A student suggests that a bigger resistor should be used to increase the time constant. Explain whether this would work.	**8.7**	(2)
	The time constant would decrease…			✓
	…as $\tau \propto 1/R$.			✓
	(f)	Calculate the current 50 ms after the switch S is closed.	**8.7**	(3)
	$I = 6.0 - 6.0 \times e^{-(50/75)}$			✓
	$I = 6.0 - (6.0 \times 0.513)$			✓
	$I = 2.92$ A			✓
				17 marks

7.	A student tried to investigate the exponential fall of voltage in an inductive circuit by using the circuit in Question 6. The reverse voltage spike that you worked out in 6 (b) was sufficient to ruin the voltage sensor. So he set up a different circuit:  The components are the same as in Question 6. The time constant is 75 ms.			
	(a)	Suggest why this circuit should work better than the one in Question 6 when it's disconnected from the supply	**8.5**	(3)
	The inductor will discharge…			✓
	…through the resistor.			✓
	The maximum voltage is 12 V.			✓
	(b)	Work out the time taken for the voltage to fall from 12.0 V to 6.0 V when switch *S* is moved to the discharge position.	**8.7**	(3)
	$6 = 12 \times e^{(-t/0.075)}$			✓
	$\ln 0.5 = -t \div 0.075$			✓
	t = 52 ms			✓

	(c)	In the space below, sketch the voltage-time graph from when the switch is moved to the discharge position. Put appropriate values on your sketch graph.	**8.6**	(3)
				✓ ✓ ✓
				9 marks
				Total = 120 marks

End of Examination

Now go back and check your work